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Variability in the Astronomical Refraction of the Rising and Setting Sun

RUSSELL D. SAMPSON,[1] EDWARD P. LOZOWSKI,[2] ARTHUR E. PETERSON,[3] AND DOUGLAS P. HUBE[4]
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ABSTRACT. Observed timings of 244 sunrises and 135 sunsets from two closely spaced geographic locations in Edmonton, Alberta, were used to determine astronomical refraction for the Sun's upper limb. The observed astronomical refraction had a mean of 0°.669, a standard deviation of 0°.175, and a range between 0°.402 and 2°.081. At sunrise, the astronomical refraction had a mean of 0°.714, a standard deviation of 0°.184, and a range between 0°.402 and 2°.081. At sunset, the astronomical refraction had a mean of 0°.579, a standard deviation of 0°.108, and a range between 0°.442 and 1°.085. There is a strong annual variation of the monthly mean and the monthly standard deviation of the astronomical refraction. Both parameters reach a maximum during the coldest months for sunrises and sunsets. Abnormally large refraction events—sometimes called Novaya Zemlya solar mirages—can occur in both warm and cold months.

1. INTRODUCTION

Sunrise and sunset can be defined as the moment at which the upper limb of the Sun makes contact with a horizon of 0° altitude. Because of the local topography, the exact time of the observed sunrise or sunset may vary depending on the altitude of the local horizon. Nonetheless, the celestial coordinates and semidiameter of the Sun are known to extreme accuracy—about ±0″.001. Therefore, in principle, the time of sunrise and sunset should also be predictable to similar accuracy—about ±0.0001 s. However, this is not possible because the sunlight must pass through the atmosphere of the Earth before reaching the observer. As sunlight traverses the atmosphere, it passes through a gradient of atmospheric density and, following Snell's law, is refracted before reaching the observer. The refractive behavior of the light from astronomical objects at altitudes greater than 15° appears to be well understood (Mahan 1962). However, below this altitude, the refraction is more variable and less predictable. Consequently, the observed time of sunrise and sunset depends on the actual atmospheric density profile. Most published times of sunrise and sunset utilize a fixed value for astronomical refraction at the horizon, usually about 0°.57 (Green 1985), rather than trying to compute the actual refraction.

Schaefer & Liller (1990, hereafter SL) have compiled a large number of observed sunset astronomical refraction values. These were obtained from timing sunsets over a sea horizon at various geographic locations, using several observers over a period from 1987 June to 1989 December. Their range in refraction was 0°.234–1°.678, with a mean of 0°.563 and standard deviation of 0°.16. Seasonal variations in the magnitude of the astronomical refraction could not be easily explored in this study since the observations were made from many different locations and climates. Also, diurnal variations in refraction were not investigated since only sunsets were observed. It should be noted that any interpretation of the standard deviation should be done with caution since the distribution of astronomical refraction values is positively skewed.

In the present study (hereafter SLPH), astronomical refraction at both sunrise and sunset was observed by a single observer from two geographic locations separated by only 2 km within Edmonton, Alberta, Canada. Since the climate of Edmonton exhibits significant seasonal changes, it was possible to explore the seasonal variation in the magnitude of astronomical refraction. Edmonton has a continental climate. There are no large bodies of water nearby to moderate seasonal climate changes. The topography is relatively flat, with the foothills of the Rocky Mountains beginning approximately 100 km to the west. The climate is classified as Dfc in the Köppen classification system. The daily mean temperature for July is 17.5°C, with an average daily minimum of 12.1°C and maximum of 22.8°C, while the daily mean for January is −11.7°C with an average daily minimum of −16.0°C and maximum of −7.3°C.[5]

2. EXPERIMENTAL PROCEDURE

If the geocentric altitude of the center of the geometric Sun (i.e., the unrefracted Sun) can be found for the moment of the observed sunrise or sunset, then the astronomical refraction R_0

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[1] Physical Sciences Department, Eastern Connecticut State University, Willimantic, CT 06226; sampsonr@easternct.edu.

[2] Department of Earth and Atmospheric Sciences, 1-26 Earth Sciences Building, University of Alberta, Edmonton, AB T6G 2E3, Canada.

[3] Department of Civil and Environmental Engineering, University of Alberta, Edmonton, AB T6G 2G7, Canada.

[4] Department of Physics, 412 Avadh Bhatia Physics Laboratory, University of Alberta, Edmonton, AB T6G 2J1, Canada.

[5] Environment Canada. 2003, Canadian Climate Normals 1971–2000, http://www.msc-smc.ec.gc.ca/climate/climate_normals/index_e.cfm.

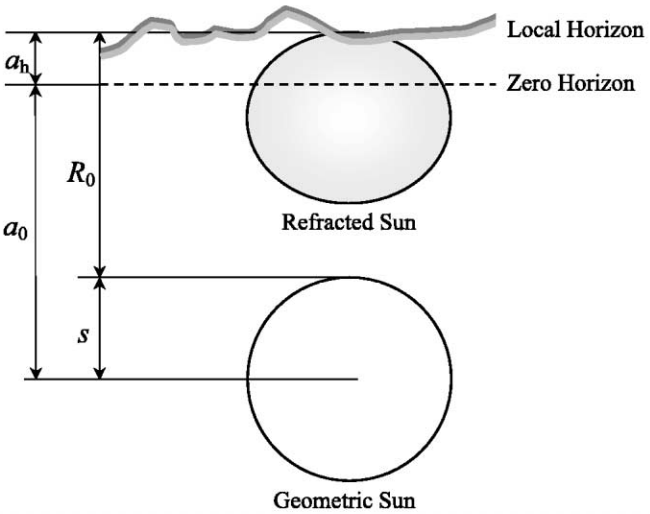

Fig. 1.—Schematic shows the essential values for the calculation of astronomical refraction at sunrise or sunset (all values in Figs. 1–4 are corrected to a horizon of 0° altitude).

can be found through the formula

$$R_0 = a_h - a_0 - s + \pi, \tag{1}$$

where s is the semidiameter of the Sun, a_h is the altitude of the observer's local horizon (at the time of sunrise or sunset), a_0 is the geocentric altitude of the center of the geometric Sun (see Fig. 1), and π is the diurnal parallax, which is taken as an average of 0°.00244. The diurnal parallax is the angular difference between the geometric and topocentric Sun at the time of sunrise or sunset and has a range of about ±0°.00008. The geocentric altitude and semidiameter of the geometric Sun were found through formulae given by Meeus (1988).

Two locations within the central part of Edmonton were used to observe the events. Both sites were elevated above the local terrain. The geographic coordinates of each location were determined to an accuracy of ±1 m in elevation and ±1″ in latitude and longitude. The first site was a 14 story apartment building with two observing stations, both at 53°31′34″ north and 113°29′14″ west, with an elevation of 687 m (apartment) and 704 m (rooftop). The second site was the rooftop of the Tory Building at the University of Alberta at 53°31′41″ north and 113°31′10″ west, with an elevation of 728 m.

The sunrise and sunsets were observed with a pair of 7 × 20 mm binoculars and a No. 14 welder's filter placed in front of the objectives. Timings were performed with a stopwatch synchronized with radio time signals (radio station WWV). The timing error was estimated to be about ±2 s, consistent with that of SL. A sketch of the azimuthal location of the sunrise or sunset was made at the time of the event. Nearby horizon landmarks such as distant power poles were used as references.

As mentioned previously, the magnitude of astronomical refraction derived from the time of sunrise or sunset depends on

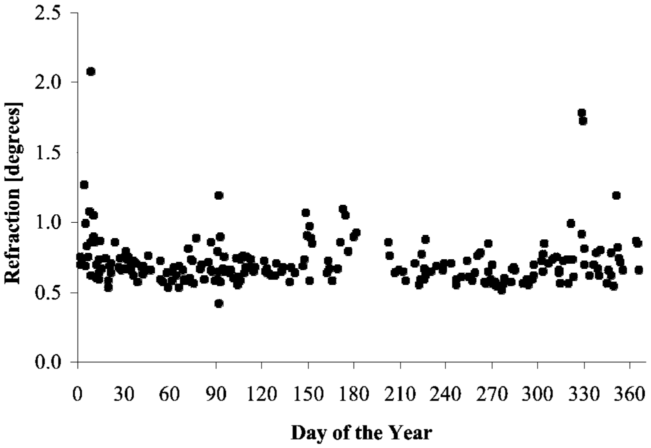

Fig. 3.—Measured astronomical refraction of sunrise events.

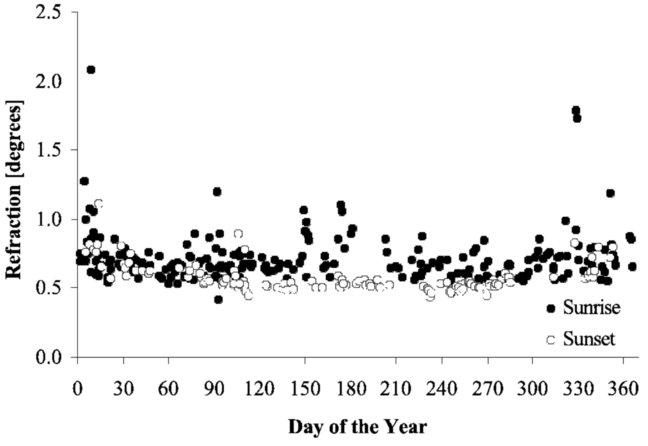

Fig. 2.—Measured astronomical refraction of both sunrise and sunset events.

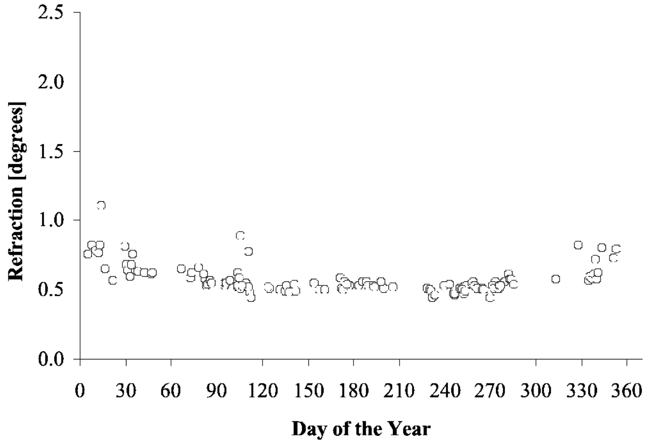

Fig. 4.—Measured astronomical refraction of sunset events.

a_h, the altitude of the local horizon at the time of the sunrise or sunset. This value was not measured at the time of the sunrise or sunset. This was done later using a theodolite capable of measuring zenith angles to a single measurement accuracy of ±0°.0017. The local horizon at the sunrise and sunset positions was found to vary from +0°.326 to −0°.246. The horizon altitude measurements were made during the middle of the day when terrestrial refraction was assumed to be minimized as a result of boundary layer mixing (Bomford 1980). To test this assumption, a distant reference marker was measured at the beginning and end of each daily measurement session. The standard deviation of all the altitude measurements of the reference marker was only ±0°.0008.

However, the altitude of the local horizon at the time of sunrise and sunset is also affected by refraction. Since the altitude measurements of the horizon mentioned above took place at a different time (around midday) than the sunrise and sunset timings, the local horizon altitude must be corrected for terrestrial refraction at the time of sunrise or sunset. A number of formulae have been developed to correct for the effects of terrestrial refraction. The authors have examined a number of these formulae and found Bomford's (1980) to be both accurate and precise (Sampson, Lozowski, & Peterson 2003). The terrestrial refraction R_t in radians is found through the formula

$$R_t = \frac{kd}{r_e}, \tag{2}$$

where d is the distance between the observer and the horizon in meters, r_e is the radius of the Earth in meters, and k is the radius of curvature of the light ray found from

$$k = \frac{252p}{T^2}\left(0.0342 + \frac{\Delta T}{\Delta h}\right), \tag{3}$$

where p is the surface pressure in millibars, T is the surface air temperature in kelvins, and $\Delta T/\Delta h$ is the atmospheric vertical temperature gradient (at the surface) in degrees per meter. The temperature and pressure for each sunrise and sunset and theodolite horizon survey were found from hourly surface measurements at a nearby meteorological station (CFB Namao, 16.5 km north of the observing sites). The hourly values were linearly interpolated to the time of the event. The vertical temperature gradient was found from rawinsonde profiles launched from Stony Plain Upper Air Station about 40 km west of the observing sites. Sunrise events used values from the morning sounding (11:15 UTC or 04:15 MST), and sunset events used the 23:15 UTC launch. In equation (2), the distance to the horizon d was estimated by solving for d in the trigonometric formula for estimating the geometric altitude (unrefracted) of the horizon a'_h:

$$\sin a'_h = \frac{(r_e + h_2)^2 - (r_e + h_1)^2 - d^2}{2d(r_e + h_1)}, \tag{4}$$

where h_1 is the elevation of the observer in meters and h_2 is the elevation of the terrain along the line of sight from the observer to the position of the sunrise or sunset. Values of d and h_2 were found using lines radiating from the observation site drawn on a topographic map. The distance to the local horizon was found when a'_h reached a maximum relative to variations in d and h_2. This value ranged from 9700 to 36,000 m.

The observed altitude of the horizon at the time of a sunrise or sunset event a_h was then determined through the expression

$$a_h = a_t - R_{t1} + R_{t2}, \tag{5}$$

where a_t is the measured altitude of the horizon (found from the theodolite), R_{t1} is the terrestrial refraction of the horizon during the time of the theodolite survey, and R_{t2} is the terrestrial refraction at the time of the sunrise or sunset event.

The average terrestrial refraction of the horizon for all the measured sunrises was found to be 0°.033, with a minimum of 0°.008 and a maximum of 0°.123. For sunsets, the average was 0°.013, the minimum was −0°.003, and the maximum was 0°.035. In future studies, it would be more efficient to measure the horizon altitude with a theodolite at the time of the sunrise or sunset event.

Before the data set could be analyzed in a consistent fashion, each event was corrected to a horizon of zero altitude. This was accomplished as follows. A plot of the astronomical refraction against the local horizon altitude a_h exhibited a linear relationship of slope m, with numerous outliers representing anomalous refraction events. In order to remove the effects of anomalous refraction and isolate the relationship between the horizon altitude and astronomical refraction, the outliers were removed by eliminating refraction values greater than a certain truncation limit L. This value was specified as follows:

$$L = \bar{R} - n\mathrm{SD}, \tag{6}$$

where $\bar{R}$ is the mean sunrise or sunset astronomical refraction, n is specified to range between 0.0 and 0.7 in increments of 0.1 (beyond 0.7, the data set becomes too sparse), and SD is the population standard deviation of the sunrise or sunset astronomical refraction. Separate corrections were derived for sunrise and sunset. The average of the eight slopes $\bar{m}$ was found to be -0.021 ± 0.003 for sunrises and -0.112 ± 0.007 for sunsets. The astronomical refraction corrected to a zero horizon R'_0 was then found through the formula

$$R'_0 = R_0 - \bar{m}a_h. \tag{7}$$

The absolute mean of the corrections was 0°.007 for sunrise

TABLE 1

Overall Astronomical Refraction with Terrestrial Refraction Correction

Source	Event	No. of Observations	Mean (deg)	SD (deg)	Minimum (deg)	Maximum (deg)
SL ………	Sunset	97	0.563	0.183	0.234	1.678
SLPH ……	Sunset	124	0.576	0.105	0.442	1.108
SLPH ……	Sunrise	234	0.714	0.184	0.420	2.081
SLPH ……	Both	358	0.666	0.174	0.420	2.081

Note.—Comparison of SL results with the corrected sunrise and sunset astronomical refraction from the present study (SLPH). The third column is the number of observations. The fifth column is the population standard deviation. The sixth and seventh columns give the range of the observed refraction.

TABLE 2

Overall Astronomical Refraction without Terrestrial Refraction Correction

Authors	Event	No. of Observations	Mean (deg)	SD (deg)	Minimum (deg)	Maximum (deg)
SL ………	Sunset	97	0.563	0.183	0.234	1.678
SLPH ……	Sunset	125	0.564	0.106	0.421	1.081
SLPH ……	Sunrise	254	0.680	0.177	0.403	2.034
SLPH ……	Both	379	0.641	0.166	0.403	2.034

Note.—Comparison of SL results with the present study, uncorrected for terrestrial refraction of the horizon. The number of observations differs from Table 1 because of missing rawinsonde data, which are needed to derive the terrestrial refraction correction.

and 0°.017 for sunset. The minimum correction was −0°.012 for sunrise and 0°.001 for sunset, while the maximum correction was 0°.016 for sunrise and 0°.055 for sunset.

3. RESULTS AND ANALYSIS

The first timing was made on 1990 December 29, and the last event was recorded on 2002 July 23. The astronomical refraction value at 234 sunrises and 124 sunsets was successfully measured. The complete data set can be found in Sampson (1994). The measured astronomical refraction is summarized in Tables 1 and 2. The original data of SL were filtered to produce a more valid comparison with the present investigation. This was done by removing all lunar and planetary events and all events with an uncertainty greater than 10 s. In SL, there appeared to be no correction for terrestrial refraction. Therefore, two tables are shown comparing the results of this study corrected and uncorrected for terrestrial refraction of the local horizon.

The mean and variability of the astronomical refraction appear to be less at sunset than at sunrise. The mean corrected sunset astronomical refraction is 0°.138 less than the mean sunrise refraction (19.4%). The mean sunset refraction in SL is very similar to this study. Their value is 0°.013 (2.3%) less than our mean *corrected* sunset refraction and 0°.001 (0.2%) less than our *uncorrected* value. The population standard deviation of SL is much greater than that for our corrected sunset values. Ours was 0°.078 (74.6%) less than that of SL.

The variation in astronomical refraction with time of year was also examined and is displayed in Figures 2–5. Previous computer simulations have suggested that astronomical refraction could vary seasonally (Sugawa 1955). Sugawa performed

TABLE 3

Observed Monthly Astronomical Refraction

Parameter	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Both												
Number ……………	43	30	37	47	28	21	14	20	41	29	21	27
Mean (deg) …………	0.801	0.675	0.640	0.634	0.656	0.676	0.618	0.606	0.572	0.587	0.828	0.704
SD (deg) ……………	0.252	0.054	0.091	0.135	0.149	0.182	0.137	0.110	0.093	0.057	0.329	0.132
Minimum (deg) ……	0.538	0.568	0.528	0.420	0.482	0.502	0.507	0.442	0.449	0.512	0.564	0.548
Maximum (deg) ……	2.081	0.788	0.890	1.194	1.067	1.102	0.929	0.878	0.850	0.722	1.788	1.192
Sunrise												
Number ……………	33	22	27	27	20	12	5	13	19	20	18	18
Mean (deg) …………	0.808	0.685	0.659	0.686	0.715	0.785	0.768	0.666	0.647	0.603	0.857	0.720
SD (deg) ……………	0.278	0.052	0.096	0.134	0.136	0.172	0.127	0.088	0.085	0.058	0.345	0.148
Minimum (deg) ……	0.538	0.568	0.536	0.420	0.568	0.578	0.644	0.557	0.553	0.517	0.564	0.548
Maximum (deg) ……	2.081	0.788	0.890	1.194	1.067	1.102	0.929	0.878	0.850	0.722	1.788	1.192
Sunset												
Number ……………	10	8	10	20	8	9	9	7	22	9	3	9
Mean (deg) …………	0.777	0.647	0.590	0.564	0.509	0.531	0.534	0.496	0.507	0.550	0.658	0.672
SD (deg) ……………	0.143	0.051	0.046	0.102	0.020	0.029	0.019	0.036	0.029	0.034	0.143	0.091
Minimum (deg) ……	0.568	0.593	0.528	0.443	0.482	0.502	0.507	0.442	0.449	0.512	0.571	0.578
Maximum (deg) ……	1.108	0.755	0.661	0.894	0.538	0.586	0.560	0.543	0.563	0.613	0.823	0.801

Note.—Summary of the monthly results including the number of observations, monthly mean astronomical refraction, standard deviations, and minimum and maximum monthly values.

TABLE 4

ANOMALOUS ASTRONOMICAL REFRACTION EVENTS

Date	Time (UTC)	T (°C)	p (mbar)	dT/dh (°C m^{-1})	a_h (deg)	R'_0 (deg)
Sunrises						
1991 Jan 6	15:44:56	−29.3	945.0	0.0481	0.203	1.273
1991 Jan 9	15:45:01	−30.1	934.7	0.0440	0.203	1.075
1991 Jan 10	15:35:54	−24.9	934.0	0.0880	0.203	2.081
1991 Apr 1	13:04:30	2.2	926.3	0.0563	0.089	1.194
1991 May 29	11:08:32	10.8	926.3	0.0437	−0.232	1.067
1991 Dec 17	15:42:10	−11.3	935.6	0.0536	0.205	1.192
1992 Jun 22	10:57:50	11.1	937.5	0.0482	−0.246	1.102
1992 Jun 24	10:59:10	9.3	935.0	0.0712	−0.246	1.052
1993 Jan 14	15:41:20	−27.9	934.1	0.1742	0.215	1.051
1993 Nov 25	15:10:14	−14.3	944.4	0.1586	0.209	1.788
1993 Nov 26	15:12:00	−8.1	935.6	0.0456	0.209	1.728
Sunsets						
1993 Jan 14	23:49:23	−18.4	927.2	0.0175	−0.096	1.108

NOTE.—The times, astronomical refraction values, and meteorological conditions for anomalous events recorded in this study. The third and fourth columns are the surface temperature and pressure. The fifth column is the surface vertical temperature gradient in which a positive value corresponds to an inversion. The sixth column is the measured altitude of the horizon, uncorrected for terrestrial refraction. The final column is the astronomical refraction, corrected to a zero horizon and corrected for terrestrial refraction of the horizon.

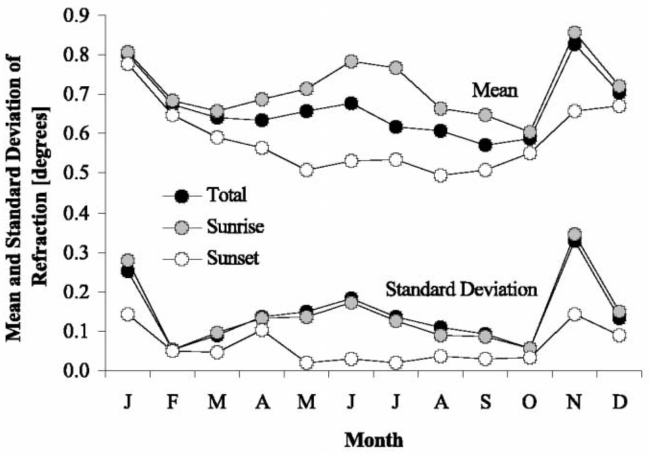

FIG. 5.—Mean monthly astronomical refraction and standard deviation of the astronomical refraction.

numerical ray tracing for altitudes of 5°–85° through monthly mean atmospheric density profiles. Temperature and pressure data were obtained from 323 profiles produced from rawinsondes launched from Sendai, Japan, at 0 and 12 hr Japan Standard Time. On the basis of these numerical simulations, Sugawa deduced that the greatest astronomical refraction occurs during the colder months, with a maximum in December and a minimum in August. The maximum refraction was 13% greater than the minimum.

In order to explore these seasonal variations, our astronomical refraction values were placed into 12 equal bins of 30.4 days. These bins were then labeled successively with the names of the 12 months. For each bin, the mean refraction and the population standard deviation were calculated and plotted (see Table 3 and Fig. 5). It is apparent from these graphs that there is a systematic difference between the mean refraction at sunrise and at sunset, particularly in the summer. The mean sunrise refraction is 0°.714, while the mean sunset refraction is 0°.576, a difference of 24%. In the summer months (June, July, and August), the difference rises to 40% (0°.730 for sunrise and 0°.522 for sunset).

Our observed monthly mean sunset astronomical refraction appears to confirm the model-predicted results of Sugawa, with a minimum in warmer months and a maximum in colder months. The maximum (0°.777 in January) is 57% greater than the minimum (0°.496 in August). During the original study, only one sunset was observed during the month of July. In the summer of 2002, almost 10 years after the last observation in the original study, a series of eight sunsets was observed from the University of Alberta site. The fact that these values are consistent with the original observations made a decade earlier suggests that there has been no significant shift in astronomical refraction.

The difference between the mean sunrise and mean sunset astronomical refraction was relatively small in the colder months (January, February, October, and December). For the rest of the year, the sunrise refraction is consistently greater than the sunset refraction, reaching a maximum difference of 0°.254 in June. The mean sunrise astronomical refraction also exhibits a narrow peak of 0°.857 in November and a broader secondary peak in July of 0°.768.

The monthly standard deviation of the astronomical refraction also appears to follow the seasonal trend exhibited by the mean astronomical refraction. The greatest variability for both sunrise and sunset occurred in the November and January bins. The relatively low value in December appears to be anomalous. A more detailed examination into the weather of the period may reveal the mechanism behind this result, but it is beyond the scope of this study. The standard deviation is consistently greater for sunrise than sunset. One of the most striking features of the sunset standard deviation plot is the interval from May through October, where the values are not only at a minimum but exhibit a high degree of stability.

More data will be necessary to confirm the seasonal behavior of the astronomical refraction. Many of the monthly bins, especially for sunsets, contain fewer than 10 data points.

On rare occasions, the Sun appeared to rise much earlier or set much later than predicted by such publications as the Tables of Sunrise, Sunset, and Twilight (USNO 1962). In our study, the sunrise of 1991 January 10 was almost 12 minutes early. This phenomenon is known as the Novaya Zemlya solar mirage (Lehn 1979). It appears to be caused by a geographically extensive temperature inversion within the boundary layer of the atmosphere. The resulting vertical density profile causes the sunlight to be ducted around the curvature of the Earth. For the purpose of this study, we defined anomalously large astronomical refraction to be an event with refraction greater than 1°. A total of 12 anomalous events were recorded (2.9%). The refraction and meteorological data for these events appear in Table 4.

The majority of the anomalous events took place in the cold months. Nine of the 12 events occurred between November 1 and April 30, with January having five events. At the time of the events, the average surface temperature was −10.9°C, and all events occurred with surface-inversion conditions. Surface inversions tend to form with overnight surface radiative cooling through a dry atmosphere. Typically, they persist into the early morning, even after sunrise. Even though most of the events took place in the cold parts of the year, the data suggest that the Novaya Zemlya solar mirage may not be an exclusively cold-weather or polar phenomenon. Four of the events occurred with a surface temperature greater than 0°C. One event took place 2 days after the summer solstice.

4. DISCUSSION AND CONCLUSIONS

Astronomical refraction of the rising and setting Sun has been successfully measured from a terrestrial horizon using a suitable correction for the effects of terrestrial refraction. A quantitative comparison with previous observations was possible once the data had been adjusted to a zero horizon. The mean sunset refraction was very similar for both SL and SLPH. However, SL exhibited a larger standard deviation. The SLPH results show a distinct difference between astronomical refraction at sunrise and sunset. Sunrise refraction is generally greater and exhibits more variability. The seasonal variation of the observed sunset refraction in this study qualitatively matches the predictions of Sugawa, with a maximum occurring in the colder months. However, the seasonal behavior of the observed sunrise astronomical refraction did not follow Sugawa's predictions as well as the observed sunset data. This may be because the lower atmosphere is better mixed during the day as a result of solar heating leading to a dry adiabatic lapse rate in the boundary layer. Anomalously large astronomical refraction events—the Novaya Zemlya solar mirage—occurred about 3% of the time and were an order of magnitude more common at sunrise than sunset. Although they were more common in cold months, they were also observed in warm months.

The behavior of astronomical refraction at sunrise and sunset appears to be variable but not at all random. Application of astronomical refraction corrections to such areas as archaeoastronomy and navigation should now consider that a fixed value is no longer suitable and that climatic trends can and should be employed. Further work in this area could concentrate on such areas as the variation of seasonal trends with geographic location and relating refraction to commonly available meteorological data.

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